TRINITY COLLEGE

Mathematics Specialist Unit 1&2
Test 1 2018

Section 1 Calculator Free

Combinatorics

STUDENT'S NAME Solutions

DATE: Thursday 22 February **TIME:** 15 minutes **MARKS**: 15

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (4 marks)

The letters of the word *APPLE* are rearranged in a line. Determine:

(a) the total number of possible arrangements [2]

$$\frac{5!}{2!} = \frac{120}{2}$$

$$= 60$$

(b) the total number of arrangements with the letters A and E are adjacent. [2]

AE _ _ _

$$\Rightarrow \frac{4! \times 2!}{2!}$$

$$= 24$$

2. (9 marks)

For the following question, leave your answers in combination and/or factorial notation.

A bookshelf contains 15 books; 5 Mathematics textbooks, 2 Dictionaries, 3 Dan Brown novels and 5 Phillipa Gregory novels.

How many ways can the books be arranged if:

(a) There are no restrictions? [1]

$15!$

(b) All the novels must be together? [2]

Novels ..... $\Rightarrow 8! \times 8!$

(c) There must be a dictionary at each end? [2]

2 ..... 13! ..... 1 $\Rightarrow 2 \times 13!$

(d) The Mathematics textbooks must be in the very middle of the shelf? [2]

..... Maths 5! ..... $\Rightarrow 5! \times 10!$

(e) The Mathematics textbooks must not be together? [2]

no restrictions − maths together

$\Rightarrow 15! - 5! \times 11!$

3. (2 marks)

Determine the value of ${}^{10}C_5$ given ${}^{10}C_6 = 210$ and ${}^{11}C_6 = 462$

From Pascal's triangle ${}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$

1
1 1
1 2 1
1 3 3 1

$\Rightarrow {}^{10}C_5 + {}^{10}C_6 = {}^{11}C_5$

$\Rightarrow {}^{10}C_5 + 210 = 462$

$\Rightarrow {}^{10}C_5 = 252$

TRINITY COLLEGE

Mathematics Specialist Unit 1&2
Test 1 2018

Section 2 Calculator Assumed

Combinatorics

STUDENT'S NAME _____

DATE: Thursday 22 February **TIME:** 35 minutes **MARKS**: 34

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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4. (7 marks)

Passwords are to be created using the letters of the alphabet (case sensitive) and the digits 0 to 9 inclusive.

(a) Write mathematical expressions for the number of possible passwords if passwords consist of 8 characters and:

(i) no character can be used more than once. [2]

52 letters + 10 digits = 62 characters

$\Rightarrow {}^{62}C_8 \times 8!$

(ii) repetition of characters are permitted. [2]

62^8

(b) The length of the password is determined by the number of characters in the password. Determine the minimum length of a password if repetition of characters is not permitted and the number of possible passwords must exceed 1×10^{15} [3]

Trial & error

${}^{62}P_8 = 1.36 \times 10^{14}$

${}^{62}P_9 = 7.36 \times 10^{15}$

∴ passwords need to have a minimum of 9 characters

5. (8 marks)

Mr Jamieson is organising his yearly trip to donate blood. Mr Jamieson will need one bus driver, two teachers (other than himself), and twelve students to make the trip worthwhile. Mr Jamieson has been inundated with volunteers and has five bus drivers to choose from, seven teachers (other than himself) to choose from and 25 students who would like to go.

How many possible groups can Mr Jamieson take if:

(a) There are no restrictions? [1]

$${}^{5}C_{1}\,{}^{7}C_{2}\,{}^{25}C_{12} = 546\,031\,500$$

(b) Mr Presser, one of the teachers who has volunteered, must go? [1]

$${}^{5}C_{1}\,{}^{1}C_{1}\,{}^{6}C_{1}\,{}^{25}C_{12} = 156\,009\,000$$

(c) Ben (a student volunteer) must go? [1]

$${}^{5}C_{1}\,{}^{7}C_{2}\,{}^{1}C_{1}\,{}^{24}C_{11} = 262\,095\,120$$

(d) Michael (a student) will only go if Ayden (also a student) goes? [3]

M & A or $\bar{M}$ & A or $\bar{M}$ & $\bar{A}$

$${}^{5}C_{1}\,{}^{7}C_{2}\,{}^{1}C_{1}\,{}^{1}C_{1}\,{}^{23}C_{10} + {}^{5}C_{1}\,{}^{7}C_{2}\,{}^{1}C_{0}\,{}^{1}C_{1}\,{}^{23}C_{11} + {}^{5}C_{1}\,{}^{7}C_{2}\,{}^{1}C_{0}\,{}^{1}C_{0}\,{}^{23}C_{12}$$

$= 404\,063\,310$

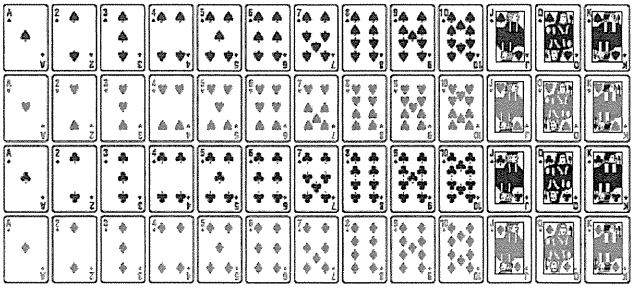
(e) Mr Presser or Ben will go? [2]

Mr P + Ben − Both

$156\,009\,000 + 262\,095\,120 - {}^{5}C_{1}\,{}^{1}C_{1}\,{}^{6}C_{1}\,{}^{1}C_{1}\,{}^{24}C_{11}$

$= 343\,219\,800$

6. (7 marks)



A standard deck of cards is shown right:

A hand consists of 5 cards.

Determine the number of hands possible where:

(a) there are no restrictions [1]

$^{52}C_5 = 2\,598\,960$

(b) you have all the same suit and the cards 10, J, Q, K and A (royal flush) [2]

$^4C_1\,{}^1C_1\,{}^1C_1\,{}^1C_1\,{}^1C_1\,{}^1C_1 = 4$

(c) any five cards of the same suit (flush) [2]

Suit 5 cards

$^4C_1 \times {}^{13}C_5 = 5148$

(d) three cards of one value and a pair of another value (full house) [2]

$^{13}C_1 \times {}^4C_3 \times {}^{12}C_1 \times {}^4C_2 = 3744$

7. (3 marks)

The letters of the word *YELLOW* are rearranged in a line. Determine the total number of 3 letter "words" that can be formed.

No Ls or 1 L or 2 Ls

$^1C_0\,{}^1C_0\,{}^4C_3 \times 3! + {}^1C_1\,{}^1C_0\,{}^4C_2 \times 3! + \dfrac{{}^1C_1\,{}^1C_1\,{}^4C_1 \times 3!}{2!}$

$= 24 + 36 + 12$

$= 72$

8. (5 marks)

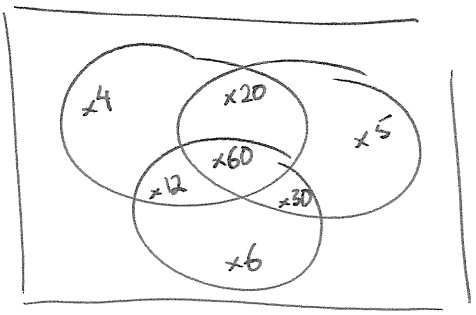
Determine the number of positive integers between 1 and 240 inclusive that are **not** divisible by at least one of the integers 4, 5 or 6.

Multiples of 4, 5 or 6

$= \frac{240}{4} + \frac{240}{5} + \frac{240}{6}$

$= 60 + 48 + 40$

$= 148$



Multiples of 12, 20 or 30

$= 20 + 12 + 8$

$= 40$

Multiples of 60

$= 4$

∴ total number of integers divisible $= 148 - 40 + 4$

$= 112$ integers

∴ total number <u>not</u> divisible $= 240 - 112$

$= 128$

9. (4 marks)

Solve for n if $12({}^{n}C_{2}) = {}^{2n}C_{3}$

$$\Rightarrow \quad 12\left(\frac{n!}{(n-2)!\,2!}\right) = \frac{(2n)!}{(2n-3)!\,3!}$$

$$\Rightarrow \quad \frac{6 \times n \times (n-1) \times (n-2)\cdots}{(n-2)\cdots} = \frac{2n \times (2n-1)(2n-2)(2n-3)\cdots}{6 \times (2n-3)\cdots}$$

$$\Rightarrow \quad 36n(n-1) = 2n(2n-1)(2n-2)$$

$$\Rightarrow \quad n = 0, 1, 5$$

$$\therefore \quad n = 5$$